Course Digest, MATH 291, Fall 2025

Week 05 Monday 09/15 and Wednesday 09/17

Read 2.1, 2.2, 2.3

Monday Today was a guest lecture by Professor Emily Witt. The focus of our class was on matrix algebra. First, we defined what it means to multiply a scalar and a matrix (which can always be done), and to add or subtract two matrices (which can only be done if they have the same size). We used these operations to solve equations for an unknown matrix and investigated the basic properties of these operations. Next, we defined the *transpose* of an $m \times n$ matrix A as the $n \times m$ matrix A^{T} whose rows are the columns of A. We established some basic properties of the transpose, and then solved some matrix equations involving it. Finally, we defined what it means to multiply a $m \times n$ matrix by a vector with n entries.

Wednesday Today, we started with a quiz. Next, we recalled what it means to multiply a $m \times n$ matrix A and a vector $\mathbf{v}$ with n entries, with the analogy that the recipe $A\mathbf{x}$ consists of ingredients, the columns of A, and the proportion of each ingredient coming from the entries of $\mathbf{x}$. We went through several examples, and then saw that when A has size $1 \times n$, then $A\mathbf{x}$ corresponds to the dot product of the vector whose coordinates are the entries of A, with $\mathbf{x}$. Next, we defined the $n \times n$ identity matrix I, and showed that for any vector $\mathbf{v}$ with n coordinates, $I\mathbf{v} = \mathbf{v}$, motivating the name "identity." After this, we saw that every system of linear equations can be written as a matrix equation of the form $A\mathbf{v} = \mathbf{w}$. Finally, we stated some basic properties of multiplication of matrices and vectors, and proved the first one.

Week 04 Monday 09/08 and Wednesday 09/10

Read 1.3

Monday Today was a guest lecture by Chris Wong. We also had our first quiz of the semester. As for the lecture, our focus remained on homogeneous systems. To start, we recalled the following **Theorem**: Three points, not all on a line, determine a circle. This is a theorem known to the ancient Greeks, and we went over an example of how to write down the formula of a circle passing through three points, not all on a line. The rest of the lecture was spent working in groups on the first Honors Exploration project. In this project, we will consider applications of homogeneous systems to chemistry, and will also produce a proof (i.e., a logical step-by-step argument) of the above theorem from geometry.

Wednesday Today was another guest lecture by Chris Wong. We started by considering the interesting situation when a homogeneous system depends on a parameter. We worked out a pair of examples of this, and students will gain more practice in this week's online homework. After this, the rest of the lecture was again spent working in groups on the first Honors Exploration project.

Week 03 Monday 09/01 and Wednesday 09/03

Read 1.3

Monday No lecture. Happy Labor Day!

Wednesday Our focus was on *homogeneous systems of linear equations*. We recalled the definition, and then observed the following: For a homogeneous system, the sum of two solutions is a solution, as is the scaling of a solution. More succinctly, a linear combination of solutions to a homogeneous system is also a solution to that system. We introduced the concepts of a *leading variable* and a *free variable*, both of which can be read from the reduced echelon form of the system, and went over multiple examples. In each example, we identified the so-called *basic solutions* to a homogeneous system.

Week 02 Monday 08/25 and Wednesday 08/27

Read 1.2, 1.3

Monday We recalled the *elementary row operations* used to simplify augmented matrices, and got more practice applying them. After evaluating the steps we used to simplify, we introduced the *row echelon form*, or simply *echelon form* for short, of an augmented matrix. We presented (non)examples, and saw that an augmented system has (infinitely) many echelon forms. To address this, we introduced the *reduced row echelon form*, or simply *reduced echelon form* for short, and stated the following **Theorem**: Every augmented matrix can by transformed, by means of elementary row operations, to a *unique* row echelon form. We then outlined the process for doing this, which is referred to as the *Guassian elimination* algorithm. We spent the rest of class going over examples of Guassian elimination.

Wednesday We started by going over further examples of Guassian elimination. After this, we introduced the notation of a *linear combination* of vectors (all of like size). We then went over examples, in which we saw that a particular vector was (or was not) a linear combination of a given list of vectors. We tried to formalize this by asking the following **Question**: Given a list of vectors, all of like size, how can we tell whether some other vector is a linear combination of these? We reasoned that we could address this question by translating it into a system of linear equations. We then went over examples.

Week 01 Monday 08/18 and Wednesday 08/20

Read Nicholson: 1.1, 1.2

Monday Welcome to Math 291! The first portion of lecture was spend on the syllabus and introductions. After that, we introduced *systems of linear equations*, and went over examples. We discussed the geometric perspective for solutions to such systems, especially the case of systems in two and three variables. We concluded lecture by presenting the *Fundamental Trichotomy of Linear Systems*, which states that any such system has either no solution, a unique solution, or infinitely many solutions.

Special assignment: Read our syllabus, and introduce yourself to me via email. Do this ASAP, and please address the following.

- Your major/minor, reasons for registering, and goals for this course.
- Your math background.
- Any future aspirations involving math. For instance, do you plan to attend graduate school, or pursue a career, in a math-adjacent area?
- Any personal facts you would like to share. For example, I discussed my family, hometown, educational background, hobbies, and pets.
- Any personal circumstances that might impact your performance. -
- A recent photo of yourself (I ask to help me identify you, but please ignore this one if you are so inclined).
- Pet photos are welcome!
- Title your message `[math-291] Introduction`. Make certain that your message conforms to the email policies described in our syllabus.

Wednesday We introduced the so-called *elementary operations* used for simplifying systems of linear equations. To simplify our work, we introduced the *augmented matrix* associated to a system, and then introduced the corresponding *elementary row operators*. These operations are (1) Swap a pair of rows, (2) scale a row by a nonzero constant, and (3) add, or subtract, a multiple of a row from another. We ended by practicing using the elementary row operations to simplify, and solve, various systems of linear equations.