

MIDTERM 02 CONCEPTUAL REVIEW

EXAM DETAILS

- Length** 5-6 problems, many of which may have multiple parts.
Material All topics covered so far this semester, with an emphasis on §2.3, §2.4, §2.5, §3.1, §3.2, a bit of §3.3
Aids One standard-sized (3×5 inch) notecard is allowed, but **no calculators are allowed**.

1. INVERSES OF MATRICES

- (1) What is the definition of the inverse of a matrix? How can you verify that matrices are inverses of each other?
2.4: 1, 28abd, 9h
- (2) How do you calculate the inverse of a square matrix? **Quiz 3 2.4: 2, 16, 17**
- (3) What is the cancellation property for matrices? In other words, when does the equation $AB = AC$ force that $B = C$? What does this have to do with invertible matrices?
- (4) Can you use inverses of matrices to solve equations involving matrices? To solve systems of equations? **Quiz 3 2.4: 3, 4, 5**
- (5) What does the phrase “*the following are equivalent*” mean in our big theorem? Have you memorized this theorem, and do you know how to use it? **2.4: 9a Quiz 5**
- (6) What are the basic properties of inverses? Can you use these properties to solve basic equations involving matrices?
2.4: 5, 9bd, 10b, 12, 24

2. ELEMENTARY MATRICES

- (1) What is an elementary matrix? How are elementary matrices related to elementary row operations? **2.5: 1**
- (2) How can you turn a single elementary row operation $A \rightarrow B$ into an equation involving A, B , and an elementary matrix?
Quiz 4 2.5: 2, 3a
- (3) Can you *quickly* compute the inverse of an elementary matrix? Why is this easier than computing the inverse of an arbitrary matrix? **Quiz 4**
- (4) Can you *quickly* compute the product of an elementary matrix and another matrix? **Quiz 4**
- (5) If you can perform a sequence of row operations to turn A into B , then how are A and B related in terms of elementary matrices? **2.5: 7**

- (6) Can you factor an invertible matrix into a product of elementary matrices? **2.5: 8**

3. DETERMINANTS

- (1) What is the determinant of a 2×2 matrix? How does this fit into the definition of the determinant of a 3×3 matrix?
3.1: 1abcdefghij
- (2) More generally, what is the definition of the determinant of a $n \times n$ matrix for $n \geq 3$? **3.1: 1klmnop**
- (3) In which ways can one *expand* to compute determinants? I.e., must one always expand across the first row? **3.1: 2**
- (4) How do determinants change after performing a single row operation? After performing a sequence of row operations? **3.1: 5, 6, 7, 8, 16ab, 17 Quiz 5**
- (5) What easy ways are there to tell if the determinant of a square matrix is zero? **3.1: 2**
- (6) What are the basic properties of determinants with respect to the operations that appear in matrix algebra? For example, what is the determinant of a product of matrices? Of the transpose of a matrix? Of the inverse of a matrix? **3.1: 9, 13**
- (7) Can you use these properties to help solve equations involving determinants? **3.2: 3, 4, 5, 6**
- (8) What do determinants have to do with invertible matrices?
3.2: 2

4. BASICS OF EIGENVALUES AND EIGENVECTORS

This topic will be emphasized less than the others listed here.

- (1) What is the definition of an eigenvalue & eigenvector?
- (2) Can an eigenvector be zero? Can an eigenvalue be zero?
- (3) How do you compute the eigenvalues of a square matrix? What is the *characteristic equation* associated to a square matrix?
- (4) Given an eigenvalue, how to do compute all of the associated eigenvectors? What is a basic eigenvector
- (5) Computing eigenvectors and eigenvalues takes some time, but it is easy to check your work. What do I mean by this?