Course Digest, MATH 590, Fall 2026

Week 03 Tuesday 09/08 and Thursday 09/10

Read FIS: 1.4, 1.5

Practice FIS: 1.4: 3, 4, 5 1.5: 1abdef, 2abcd

Tuesday We defined the concept of a *linear combination*, and covered many examples in which we checked whether a given vector was a linear combination of other fixed vectors. The idea was to translate everything to a system of linear equations, and then apply techniques from MATH 290 to see if solutions exist. After this, we defined the *span* of a collection of vectors. We covered examples, and proved the following **Theorem**: If $x_1, \ldots, x_n \in \mathsf{V}$, then $\operatorname{span}(x_1, \ldots, x_n)$ is the smallest subspace of V that contains $x_1, \ldots, x_n$. Before the proof, we carefully explained what we mean by *smallest* here.

Thursday We had our second quiz of the semester. After that, we defined what it means for vectors in V to be *generators* for V, and went over lots of examples. After this, motivated by questions of uniqueness of expressions in terms of generators, we defined what it means for vectors to be *linearly independent*. Recall that $x_1, \ldots, x_n \in \mathsf{V}$ are linearly independent if the only way to realize the zero vector as a linear combination of these vectors is the trivial way, where every coefficient is zero. We proved that this is the same as saying that no x_i is a linear combination of the others, which justifies the use of *independent* in this context. We concluded with more examples

Week 02 Tuesday 09/01 and Thursday 09/03

Read FIS: 1.2, 1.3

Practice 1.2: 1, 2, 4 1.3: 8, 10 (take $n = 3$, if you want)

Tuesday We presented many examples of vector spaces; this took most of lecture. We proved basic facts, including the fact that $0x = 0$ for every vector x, where 0 is the zero scalar (constant) on the left, and the zero vector on the right. We defined a *subspace* and revisted earlier examples to exhibit subspaces. We discussed the difference between a *subset* and *subspace*, and ended by describing how to verify that a subset of a vector space is a subspace.

Thursday We started with our first quiz of the semester. After this, we recalled the three criteria that must be verified to prove that W of a vector space V is a subspace: 0) W must contain the zero vector, 1) W must be closed under addition, 2) W must be closed under scaling. We then spent the rest of the lecture carefully examining many (non)examples.

Week 01 Tuesday 08/25 and Thursday 08/27

Read Skim Appendices A,B,C,D in FIS

Practice None this week

Tuesday Welcome to Math 590! We spent the first part of the lecture on the syllabus and introductions. We then introduced basic set notation, and noted that there are two basic types of algebraic objects in MATH 290/291, namely, scalars and vectors. The rest of lecture was focused on properties of scalars, and we introduced the notion of a *field*.

Thursday We focused on the field axioms. We saw that each of $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ is a field, and that $\mathbb{Z}$ is not a field. We also spent some time discussing *finite fields* via "clock arithmetic" with a clock divided up into a prime number of hours. We then proved various uniqueness statements in fields (e.g., the uniqueness of the zero constant, and the uniqueness of negatives). We then introduced the *vector space axioms*, and considered the vector space $\mathsf{V} = \mathbb{F}^n$, which is the set consisting of all lists of length/size n with entries in $\mathbb{F}$. Of course, this is a simple generalization of the sets $\mathbb{R}^2, \mathbb{R}^3$ considered in Calculus.