Week 03 Tuesday 09/08 and Thursday 09/10

Read 3.1, 4.1

Exercises 3.1: 1 (assume the field is $\mathbb{Q}$), 5, 8, 10, 4.1: 3, 4

Tuesday We defined what it means for two polynomials to be *relatively prime*, and for a polynomial to be *irreducible*. We applied Bezout's Identity to demonstrate that irreducible polynomials behave much like prime integers. This culminated in us stating, and mostly proving, the *Unique Factorization Theorem* for polynomials over a field. We then discussed the connection between roots of a polynomial and factors of the form $x - \alpha$ with α a constant. After this, we finally returned to linear algebra, and recalled how to substitute an operator for the variable in a polynonial to obtain another operator. We ended by presenting an example of an operator $T : \mathbb{R}^2 \to \mathbb{R}^2$ and a nonzero polynomial $f(x) \in \mathbb{R}[x]$ such that $f(T) = 0$, the zero operator, and stated a related result (one that we will focus on soon).

Thursday We started with our second quiz of the semester. For the lecture, we considered the following general **Question**: Given a finite dimensional V, a linear transformation $\mathsf{T} : \mathsf{V} \to \mathsf{V}$, and a vector $\mathbf{v} \in \mathsf{V}$, can we build an operator from T in a natural way that maps $\mathbf{v}$ to $\mathbf{0}$? What can we say about all such operators built from T? We addressed this by establishing the following **Theorem**: There exists a polynomial $0 \neq p(x) \in \mathbb{F}[x]$ such that $p(\mathsf{T})(\mathbf{v}) = \mathbf{0}$. Furthermore, there exists a *least such polynomial*, that is, a monic polynomial $\mu_{\mathsf{T},\mathbf{v}}(x) \in \mathbb{F}[x]$ with the property that $\mu_{\mathsf{T},\mathbf{v}}(\mathsf{T})(\mathbf{v}) = \mathbf{0}$ and such that $\mu_{\mathsf{T},\mathbf{v}}$ divides any other polynomial $p(x) \in \mathbb{F}[x]$ with $p(\mathsf{T})(\mathbf{v}) = \mathbf{0}$. This is called the *minimal polynomial of* T *with respect to* $\mathbf{v}$. We then carefully went over an example of how to compute this in a specific case.

Week 02 Tuesday 09/01 and Thursday 09/03

Read Read Cooperstein 3.1, 3.2, 4.1

Exercises Cooperstein 3.1: 1 (assume the field is $\mathbb{Q}$)

Tuesday We continued our discussion of linear transformations and matrices. We defined the vector space $\mathcal{L}(\mathsf{V}, \mathsf{W})$ and explained why $\mathcal{L}(\mathsf{V}) = \mathcal{L}(\mathsf{V}, \mathsf{V})$ is special. We stated the existence of an isomorphism between $\mathcal{L}(\mathsf{V}, \mathsf{W})$ and the vector space of $\dim(\mathsf{V}) \times \dim(\mathsf{W})$ matrices, when both of these dimensions are finite. The rest of lecture was dedicated to polynomials. We discussed how to substitute a linear transformation for a variable in a polynomial, and after introducing notation, and examples, we stated, and proved, the *Division Algorithm* for polynomials over a field.

Thursday We introduced what it means for a polynomial to divide another, and then recalled the definition of a greatest common divisor of two polynomials (not both zero). We then recalled the *Euclidean Algorithm* for computing a greatest common divisor, and went over an example. In that example, we saw that the greatest common divisor of a pair of polynomials in $\mathbb{F}[x]$ can be written as an $\mathbb{F}[x]$-linear combination of those polynomials. This provided an example of the so-called *Bezout Identity*

Week 01 Tuesday 08/25 and Thursday 08/27

Read Review Cooperstein 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 2.1, 2.2, 2.4, 2.5, 2.6

Exercises Focus on skimming the book

Tuesday Welcome to Math 790! The first portion of lecture was dedicated to the syllabus and introductions. After this, we began our review of the basic facts of linear algebra, i.e., material from MATH 590. We recalled the definition of an abstract vector space, and presented examples. We also recalled the concepts of linear independence, span, and bases. We discussed infinite dimensional vector spaces, though we focus on the finite dimensional case.

Thursday We continued our review of MATH 590 topics, with an emphasis on linear transformations and subspaces. We recalled the definition of the kernel and image (range) of a linear transformation, and what it means for a linear transformation to be an isomorphism. We recalled how to turn a linear transformation between finite dimensional vector spaces into a matrix, and the basic properties of this process.