Final Exam Conceptual Review

Exam details

The final exam is *cumulative*, but there will be an emphasis on the material since Midterm 02. This conceptual review is meant to cover that material. For topics from earlier in the semester, review the previous conceptual reviews, which are posted on the course website

Length	06-08 problems, many with multiple parts.
Material	§3.1, §3.2, §3.3, §3. 4
Aids	Two standard-sized (3×5 inch) notecards are allowed, but **no calculators are allowed**.

1. Determinants

(1) What is the determinant of a 2×2 matrix? How does this fit into the definition of the determinant of a 3×3 matrix? **3.1: 1abcdefghij**

(2) More generally, what is the definition of the determinant of a $n \times n$ matrix for $n \geq 3$? **3.1: 1klmnop**

(3) In which ways can one *expand* to compute determinants? I.e., must one always expand across the first row? **3.1: 2**

(4) How do determinants change after performing a single row operation? How do determinants change after performing a sequence of row operations? **3.1: 5, 6, 7, 8, 16ab, 17**

(5) What easy ways are there to tell if the determinant of a square matrix is zero? **3.1: 2**

(6) What are the basic properties of determinants with respect to the operations that appear in matrix algebra? For example, what is the determinant of a product of matrices? Of the transpose of a matrix? Of the inverse of a matrix? **3.1: 9, 13**

(7) Can you use these properties to help solve equations involving determinants? **3.2: 3, 4, 5, 6**

(8) What do determinants have to do with invertible matrices? **3.2: 2**

2. Basics of eigenvalues and eigenvectors

For practice computing eigenvalues and eigenvectors, review past Lyryx problems, Honors Exploration 2, and the suggested problems from the next section in this conceptual review.

(1) What is the definition of an eigenvalue & eigenvector?
(2) Can an eigenvector be zero? Can an eigenvalue be zero?
(3) How do you compute the eigenvalues of a square matrix? What is the *characteristic equation* associated to a square matrix?
(4) Given an eigenvalue, how to do compute all of the associated eigenvectors? What is a basic eigenvector
(5) Computing eigenvectors and eigenvalues takes some time, but it is easy to check your work. What do I mean by this?

3. Diagonalization and applications

(1) What does it mean for a matrix A to be diagonalizable in terms of the eigenvalues and eigenvectors of A?
(2) Given a matrix A that is diagonalizable, how can you express it as $A = PDP^{-1}$? That is, what are P and D? **3.4: 1**
(3) What is the usefulness of the expression $A = PDP^{-1}$? Can you use it to quickly read off the determinant of A?
(4) More generally, if A is diagonalizable, then how does this help you to efficiently compute A^k **3.4: 2**
(5) Can you use the ideas of diagonalization to give explicit formulas for sequences that are defined recursively? **Honors Exploration 2**