

# MIDTERM 01 CONCEPTUAL REVIEW

## EXAM DETAILS

**Length** 5-6 problems, many of which may have multiple parts.

**Material** §1.1, §1.2, §1.3 and §2.1, §2.2, §2.3.

**Aids** One standard-sized ( $3 \times 5$  inch) notecard is allowed, but no calculators are allowed.

Note: Sample problems appear in **blue** and quizzes appear in **red**.

### 1. SYSTEMS OF LINEAR EQUATIONS

- (1) What is the definition of a linear equation? Do you know examples of equations that are not linear?
- (2) What is a *solution* to a linear system? What is the trichotomy that describes the solutions to systems of linear equations?  
**§1.1: 1**
- (3) What is the augmented matrix associated to a system of linear equations? **§1.1: 7, 8**
- (4) What are the standard row operations? **Quiz 01**
- (5) What is the definition of row echelon form (REF)? Of reduced row echelon form (RREF)? What is the difference between these two definitions? **§1.2: 1, 2**
- (6) What is a leading variable? A free variable? How do they help you solve linear systems?
- (7) Can you solve linear systems? **§1.2: 3, 5, 12, Quiz 01**
- (8) Can you solve system of equations that involve unknown constants? In examples, can you tell when a system that involves unknown constants has a unique solution, no solution, or infinitely many solutions? **§1.1: 12 §1.2: 9 §1.3: 2 Quiz 03**
- (9) What does the term *linear combination* mean? Given a collection of vectors, all of the same size, how would you go about checking if some given vector is a linear combination of the vectors in your collection? **§1.3: 3, 4**
- (10) What is the definition of a homogeneous system of linear equations? What is the dichotomy that describes the solution to a system of linear equations? In this context, what does the terminology *trivial solution* refer to? **§1.3: 1**
- (11) Can you solve homogeneous systems of equations? What is a *basic solution* to a homogeneous system? What does it have to do with leading and free variables? **§1.3: 5 Quiz 02**

## 2. MATRIX ALGEBRA

- (1) When can you add and subtract matrices?
- (2) What are the basic properties of scalar multiplication for matrices, matrix addition, subtraction, and transposition? Can you solve matrix equations involving these basic operations? **§2.1: 1, 3, 4, 15**
- (3) What is the definition of multiplying an  $m \times n$  matrix by a column vector? What does it have to do with linear combinations?
- (4) Can you compute  $A\mathbf{x}$  when  $A$  is a matrix and  $\mathbf{x}$  is a column vector of the appropriate size?
- (5) What basic properties hold for the operation  $A\mathbf{x}$ ? How does it behave with respect to scalar multiplication? What are the distributive properties here? Can you use these to simplify expressions? **§2.2: 3, 4**
- (6) Given a system of linear equations, can you rewrite it in the form  $A\mathbf{x} = \mathbf{b}$ ? **§2.2: 2**
- (7) What is the definition of matrix multiplication? When can you multiply two matrices? **§2.3: 1, 2, 3**
- (8) What is the dot product of a row and column? What does this have to do with matrix multiplication?
- (9) What is the identity matrix? What properties does it satisfy?
- (10) What are the basic properties of matrix multiplication? Can you use these to simplify expressions? **§2.3: 4, 16**