Midterm 02 Conceptual Review

Exam details

Length 5-6 problems, many of which may have multiple parts.
Material All topics covered so far this semester, with an emphasis on §2.3, §2.4, §2.5, §3.1, §3.2
Aids One standard-sized (3×5 inch) notecard is allowed, but **no calculators are allowed**.

1. Inverses of matrices

(1) What is the definition of the inverse of a matrix? How can you verify that matrices are inverses of each other?
2.4: 1, 28abd, 9h
(2) How do you calculate the inverse of a square matrix? **Quiz 4**
2.4: 2, 16, 17
(3) What is the cancellation property for matrices? In other words, when does the equation $AB = AC$ force that $B = C$? What does this have to do with invertible matrices?
(4) Can you use inverses of matrices to solve equations involving matrices? To solve systems of equations? **Quiz 4 2:4 3, 4, 5**
(5) Do you understand what the phrase "*the following are equivalent*" in the context of the theorem on the Course Digest from Week 10 (Tuesday) means? Have you memorized this theorem, and do you know how to use it? **2.4: 9a Quiz 6**
(6) What are the basic properties of inverses? Can you use these properties to solve basic equations involving matrices?
2.4: 5, 9bd, 10b, 12, 24

2. Elementary matrices

(1) What is the definition of an elementary matrix? How are elementary matrices related to elementary row operations?
2.5: 1
(2) How can you turn a single elementary row operation $A \to B$ into an equation involving A, B, and an elementary matrix? **Quiz 5 2.5: 2, 3a**
(3) Can you *quickly* compute the inverse of an elementary matrix? Why is this easier than computing the inverse of an arbitrary matrix? **Quiz 5**
(4) Can you *quickly* compute the product of an elementary matrix and another matrix? **Quiz 5**

(5) If you can perform a sequence of row operations to turn A into B, then how are A and B related in terms of elementary matrices? **2.5: 7**
(6) Can you factor an invertible matrix into a product of elementary matrices? What does this have to do with row reduction? **2.5: 8**

3. Determinants

(1) What is the determinant of a 2×2 matrix? How does this fit into the definition of the determinant of a 3×3 matrix? **3.1: 1abcdefghij**
(2) More generally, what is the definition of the determinant of a $n \times n$ matrix for $n \geq 3$? **3.1: 1klmnop**
(3) In which ways can one *expand* to compute determinants? I.e., must one always expand across the first row? **3.1: 2**
(4) How do determinants change after performing a single row operation? How do determinants change after performing a sequence of row operations? **3.1: 5, 6, 7, 8, 16ab, 17 Quiz 6**
(5) What easy ways are there to tell if the determinant of a square matrix is zero? **3.1: 2**
(6) What are the basic properties of determinants with respect to the operations that appear in matrix algebra? For example, what is the determinant of a product of matrices? Of the transpose of a matrix? Of the inverse of a matrix? **3.1: 9, 13**
(7) Can you use these properties to help solve equations involving determinants? **3.2: 3, 4, 5, 6**
(8) What do determinants have to do with invertible matrices? **3.2: 2** This topic will be covered in the last lecture before Midterm 02