Week 16 Tuesday 12/10 and Thursday 12/12

Read FIS 3.2

Practice FIS 3.2: 1, 2, 4, 5 (pick a few), 6, 7 (these are repeats of last week),2.5: 2, 3abc, 4, 5

Tuesday We recalled the main theorem from last week (TFAE list with 5 items), and presented a direct proof of the implication (3) $\implies$ (4) that was written and shared with my by Tyson K. After this, we showed how to apply the ideas of the proof of that result to factor a given invertible matrix into a product of elementary matrices. We then discussed how to combine our newfound algorithm for computing the inverse of an arbitrary $n \times n$ matrix (or determining that such an inverse does not exist) and earlier methods to compute formulas for the inverse of a linear transformation. We spent the last portion of class discussing determinants, but unfortunately, we will not have time this semester for an in-depth examination of this topic. Next class, we will work in groups to establish, among other things, an explicit formula for the reflection of a point across a line.

Thursday We started the lecture by introducing the following problem: If $\mathsf{T} : \mathsf{V} \to \mathsf{V}$ is a linear transformation, and we know the matrix $[\mathsf{T}]_\gamma^\gamma$ for some basis γ for V, how to do we determine $[\mathsf{T}]_\beta^\beta$ for some other basis? To address this, we introduced the change of bases matrix $[\mathrm{I}_\mathsf{V}]_\beta^\gamma$ and $[\mathrm{I}_V]_\gamma^\beta$, where $\mathrm{I}_\mathsf{V} : \mathsf{V} \to \mathsf{V}$ is the identity transformation given by $\mathrm{I}_\mathsf{V}(x) = x$ for every $x \in \mathsf{V}$. We noted that each such change of basis matrix is the inverse of the other, so you don't need to compute both. We also proved that $[\mathsf{T}]_\beta^\beta = [\mathrm{I}_\mathsf{V}]_\gamma^\beta [\mathsf{T}]_\gamma^\gamma [\mathrm{I}_\mathsf{V}]_\beta^\gamma$. We ended the lecture, and the semester, by using this to give an explicit formula for the reflection across a line in $\mathbb{R}^2$.

Week 15 Tuesday 12/03 and Thursday 12/05

Read FIS 3.1, 3.2

Practice FIS 3.1: 1, 2, 3 FIS 3.2: 1, 2, 4, 5 (pick a few), 6, 7 FIS 2.5: 2

Tuesday We started by recalling (and essentially reproving)the following **Theorem**: If A is a matrix, then the rank of A, by which we mean the rank of the linear transformation L_A, is the maximal number of linearly independent columns of A. After this, we recalled the basics of elementary matrices, and then proved the following **Theorem**: If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is a linear transformation, and $\mathsf{S} : \mathsf{W} \to \mathsf{W}$ and $\mathsf{U} : \mathsf{V} \to \mathsf{V}$ are isomorphisms, then the rank of T equals the rank of STU. After going over the proof, we presented the matrix version of this result, namely, **Corollary**: If A is an $m \times n$ matrix, and P is an invertible $m \times m$ matrix, and Q is an invertible $n \times n$ matrix, then the rank of PAQ is the rank of A. We then discussed how this allows us to compute the rank of a matrix by applying elementary row and column operations to simplify it. Motivated by this, we considered the following **Theorem**: If A is a $m \times n$ matrix, then by applying row and column operations, A may be simplified to the form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$, and then the rank of A equals r. We then proved the following interesting **Corollary**: The rank of A and A^{tr}, the transpose of A, agree. In other words, the maximal number of linearly independent columns of A equals the maximal number of linearly independent rows of A.

Thursday We recalled the key points from the last lecture. After providing some motivation, we stated and carefully proved the following

Theorem: If A is an $n \times n$ matrix, then TFAE.

(1) A is invertible
(2) The rank of A in n.
(3) A can be simplified $A \sim \cdots \sim I$ using only row operations.
(4) A can be simplified $A \sim \cdots \sim I$ using only column operations.
(5) A is the product of invertible matrices.

Note: After class, both Avery and Tyson suggested me a more efficient way to do this! After this, we showed how to use this result to prove the following **Fundamental Algorithm**: Suppose that A is square.

(1) If we can simply $[A|I] \sim \cdots \sim [I|B]$ using only row operations, then A is invertible and $B = A^{-1}$.
(2) If such a simplification is not possible, then A^{-1} does not exist.

Week 14 Tuesday 11/26 and Thursday 11/28

Read Enjoy the break

Practice Enjoy the break

Tuesday We recalled the elementary row and column operations, and using these, introduced the concept of an elementary matrix. After going over examples, we presented the following **Theorem**: Consider a square matrix A.

(1) If $A \sim B$ is a single row operation, and E is the elementary matrix corresponding to this operation, then $B = EA$.

(2) If $A \sim B$ is a single column operation, and E is the elementary matrix corresponding to this operation, then $B = AE$.

We did not present the proof, but did cover lots of examples. After this, we described how to turn a matrix A that is $m \times n$ into a linear transformation $\mathsf{L}_A : \mathbb{R}^n \to \mathbb{R}^m$. Using this, we defined the rank of A to be the rank of this transformation, and we ended the lecture by proving the **Theorem** The rank of a matrix A is the maximal number of linearly independent columns of A. At the end of lecture, we returned Midterm 02, and discussed the scale.

Thursday Happy Thanksgiving

Week 13 Tuesday 11/19 and Thursday 11/21

Read Study for Midterm 02

Practice Complete Conceptual Review for Midterm 02

Tuesday We defined what it means for a linear transformation to be an *isomorphism*, and it what it means for two vector spaces to be *isomorphic*. After this, we recalled the main results from the last lecture, including the theorem that, for isomorphisms, "*the inverse of the matrix associated to* T *is the matrix associated to the inverse* T^{-1}". We spent the portion of the lecture discussing how to apply this to get explicit formulas for the inverses of isomorphisms. We ended by proving the following **Theorem** If V and W are finite-dimensional, then they are isomorphic if and only if their dimensions agree.

Thursday Today is Midterm 02.

Week 12 Tuesday 11/12 and Thursday 11/14

Read FIS 2.3, 2.4

Practice FIS: 2.3.2 (check if it is 1-1 and onto), 2.3.3(a), 2.3.12

Tuesday We started of by recalling how to turn a linear transformation into a matrix, given bases for the domain and target. We went over an example of how to do this for a particular function $\mathsf{T} : \mathsf{P}_2(\mathbb{R}) \to \mathsf{M}_{2\times 2}(\mathsf{R})$. After this, motivated by the situation for matrices, we proved the following **Theorem**: If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ and $\mathsf{U} : \mathsf{V} \to \mathsf{W}$ are linear, then the sum $\mathsf{T} + \mathsf{U}$ is also linear. We then recalled the basics of composition of functions, and proved the **Theorem**: If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ and $\mathsf{U} : \mathsf{W} \to \mathsf{Z}$ are linear transformations, then the composition $\mathsf{U} \circ \mathsf{T} : \mathsf{V} \to \mathsf{Z}$ is linear. We then recalled some basic facts about invertible functions, and proved the following **Theorem**: If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is linear, and also one-to-one and onto, then the inverse function $\mathsf{T}^{-1} : \mathsf{W} \to \mathsf{V}$ is also linear. Recall that the inverse function satisfies $\mathsf{T}(\mathsf{T}^{-1}(y)) = y$ for every $y \in \mathsf{W}$ and $\mathsf{T}^{-1}(\mathsf{T}(x)) = x$ for every $x \in \mathsf{V}$. These conditions played a key role in our proof.

Thursday We started the morning with Quiz 06. Next, we recalled the key highlights from last time, and then started taking a closer look at what happens when we turn linear transformations into matrices. The main result for the day was **Theorem** Suppose that $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ and $\mathsf{U} : \mathsf{W} \to \mathsf{Z}$ are linear transformations, so that the composition $\mathsf{U} \circ \mathsf{T} : \mathsf{V} \to \mathsf{Z}$ is linear. If α is a basis for V, β is a basis for W, and γ is a basis for Z, then

$$[][\mathsf{U} \circ \mathsf{T}]_\alpha^\gamma = [][\mathsf{U}]_\beta^\gamma [\mathsf{T}]_\alpha^\beta$$

After going over examples of how to use this theorem, we pointed out an important **Corollary**: Suppose that $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is one-to-one and onto, so that the inverse function $\mathsf{T}^{-1} : \mathsf{W} \to \mathsf{V}$ exists and is linear. If α is a basis for V and β is a basis for W, then $[\mathsf{T}^{-1}]_\beta^\alpha = ([\mathsf{T}]_\alpha^\beta)^{-1}$. In other words, the matrix of an inverse transformation is the inverse of the matrix of the transformation.

Week 11 Tuesday 09/03 and Thursday 09/05

Read Finish FIS 2.1, start FIS 2.2

Practice FIS 2.1: 10, 11, 12 FIS 2.2: 2, 3

Tuesday We started with a quiz. The lecture was focused how to give an efficient description of a linear transformation $\mathsf{T} : \mathsf{V} \to \mathsf{W}$. We introduced and discussed the so-called **Extrapolation Principle**: If $x_1, \ldots, x_n$ is am arbitrary basis for T, then we can recover all relevant information about T from the values of $\mathsf{T}(x_1), \ldots, \mathsf{T}(x_n)$. For instance, we saw how knowledge of the data points $\mathsf{T}(x_1), \ldots, \mathsf{T}(x_n)$ can be used to recover an explicit formula for T. We also saw that these data points can be used to tell when T is onto, the point being that $\mathsf{R}(\mathsf{T}) = \text{span}(\mathsf{T}(x_1), \ldots, \mathsf{T}(x_n))$. The story for when T is one-to-one is more interesting. In this direction, we proved the following **Theorem**: T is one-to-one if and only if $\mathsf{T}(x_1), \ldots, \mathsf{T}(x_n)$ is linearly independent. We went over this carefully, and our proof of this occupied the rest of the class.

Thursday We continued exploring how one might describe a linear transformation to a computer. Given a basis $\beta = \{x_1, \ldots, x_n\}$ for a vector space V, we described how to turn a vector in V into a concrete vector $[x]_\beta$ in $\mathbb{F}^n$. After going over examples, we asked how we might do something similar for matrices. If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is a linear transformation, and $\beta = \{x_1, \ldots, x_n\}$ is a basis for V, and $\gamma = \{y_1, \ldots, y_m\}$ is a basis for W, then we constructed the matrix $[\mathsf{T}]_\beta^\gamma$ by recording the coefficients needed to express teach transformed basis vector $\mathsf{T}(x_1), \ldots \mathsf{T}(x_n)$ in terms of the basis γ for W. We covered examples, and then defined how to scale a linear transformation, when and how to add linear transformations, and when and how to compose linear transformations.

Week 10 Tuesday 10/29 and Thursday 10/31

Read FIS 2.1, Axler 3A, 3B

Practice FIS: 1, 2-6 (practice with rank-nullity) 14abc, 17,

Tuesday We started off our lecture by recalling what it means for a function to be one-to-one (or injective), and then proved the following **Theorem**: If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is a linear transformation, then T is one-to-one (i.e., is injective) if and only if $\ker(\mathsf{T}) = \{0\}$. In examples, we showed how to use this to determine whether a concrete transformation was 1-1. After this, we defined the *range* of a linear transformation, and then proved that this is a subspace of W. We defined the *rank* of T to be the dimension of its range. After this, we proved the following **Theorem**: If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is linear, and $x_1, \ldots, x_n$ is a basis for V, then $\mathsf{T}(x_1), \ldots, \mathsf{T}(x_n)$ is a basis for $\mathsf{R}(\mathsf{T})$, the range of T.

Thursday We started off class by computing the rank and nullity (and more generally, bases for the range and kernel) of a particular linear transformation. After this, we recalled the statement of the **Rank-Nullity Theorem** If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is a linear transformation with V finite-dimensional, then $\text{rank}(T) + \text{nullity}(T) = \dim \mathsf{V}$. We immediately presented a philosophical argument for why this must be true, but postponed the actual proof for a bit, instead spending time going over examples. To start, we proved the following **Corollary**: If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is a linear transformation of finite dimensional vector spaces with $\dim \mathsf{V} = \dim \mathsf{W}$, then TFAE:

(1) T is $1-1$.
(2) T is onto.

We then went over lots of examples of how to use Rank-Nullity. The rest of the lecture was dedicated to a detailed presentation of the proof of Rank-Nullity.

Week 09 Tuesday 10/22 and Thursday 10/24

Read FIS 2.1, Axler 3A, 3B

Practice FIS: 2.1.9, 2.1.2 - 2.1.6 (for each, compute the kernel, which your book writes as $\mathsf{N}(T)$ and the nullity of T)

Tuesday We spent the first portion of class going over more examples of a linear transformation. Next, we established some basic consequences of the definition of a linear transformation, that is, we proved the following **Theorem**: If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is a linear transformation, then

(1) $\mathsf{T}(0_\mathsf{V}) = 0_\mathsf{W}$
(2) $\mathsf{T}(-x) = -\mathsf{T}(x)$ for each $x \in \mathsf{V}$.
(3) $\mathsf{T}(x - y) = \mathsf{T}(x) - \mathsf{T}(y)$.

We actually proved each of these twice, once for each of the defining conditions of a linear transformation. After this, we introduced the concept of the *kernel* of a linear transformation, and concluded lecture by proving the **Theorem**: The kernel of a linear transformation is a subspace of the domain.

Thursday We started lecture with Quiz 04. After this, we recalled the definition of the kernel of a linear transformation, which we proved last lecture was a subspace. We then defined the *nullity* of a linear transformation to be the dimension of its kernel. We spent the rest of the lecture going examples of various linear transformations. In each situation, we computed a basis for the kernel, and in doing so, the nullity of the linear transformation.

Week 08 Tuesday 10/15 and Thursday 10/17

Read FIS 2.1, Axler 3A

Practice FIS: 2.1.2 - 2.1.6 (but only do the part of proving that each transformation is linear)
Axler: 3A: 1

Tuesday Fall Break 2024!

Thursday Today's lecture was focused on linear transformation. After introducing function notation, we gave the following **Definition**: A function $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ between two vector spaces is a *linear transformation* (or *linear* for short) if it satisfies the following two conditions.

(1) $\mathsf{T}(x + y) = \mathsf{T}(x) + \mathsf{T}(y)$ for every $x, y \in \mathsf{V}$.
(2) $\mathsf{T}(\lambda x) = \lambda \mathsf{T}(x)$ for every $\lambda \in \mathbb{F}$ and $x \in \mathsf{V}$.

We then went over many examples, and spent the last 20 minutes, or so, of lecture going over the outcome of Midterm 01. **NOTE**: If you are reading this, and would like more practice with row reduction, or any other techniques from MATH 290, please let me know.

Week 07 Tuesday 10/08 and Thursday 10/10

Read FIS: 1.6 Axler: 2B

Practice FIS: 1.6: 2, 3 (but use the "size is right" theorem), 4, 5, 14, 15 (with only $n = 3$). Axler 2B: 7 (use "size is right" theorem).

Tuesday We started the lecture by proving the following result **Theorem**[Size is right] If V is n-dimensional and $v_1, \ldots, v_n \in \mathsf{V}$, then TFAE:

(1) $v_1, \ldots, v_n$ is independent.
(2) $v_1, \ldots, v_n$ generate V.

The proof involved applying the Replacement Theorem in two different ways. After this, we turned our attention to subspaces, and proved the following **Theorem**: Suppose that W is a subspace of V and that $\dim \mathsf{V} = n$.

(1) $\dim \mathsf{W} \leq \dim \mathsf{V}$, and equality holds if and only if $\mathsf{W} = \mathsf{V}$.
(2) Every basis for W can be extended to a basis for V. In other words, given a basis for W, it is possible to add vectors to it, and in this way, obtain a basis for V.

Again, our tool for proving these was the Replacement Theorem. The rest of class was dedicated to examples of computing bases and dimension for various subspaces.

Thursday Today was Midterm 01.

Week 06 Tuesday 10/01 and Thursday 10/03

Read FIS: 1.6 Axler: 2B

Practice Axler 2A: 8, 10, 14, 2B: 2 FIS 1.6: 1defg, 2, 3

Tuesday Today's entire lecture was dedicated to the proof of the Replacement Lemma.

Thursday Today, we defined what it means for a collection of vectors to be a *basis* for a vector space. After this, we introduced the vector space $\mathsf{M}_{m\times n}(\mathbb{R})$ of $m\times n$ matrices with real entries, so that we may refer to it in examples going forward. We described the *standard bases* of $\mathbb{R}^n, \mathsf{P}_n(\mathbb{R})$, and $\mathsf{M}_{m\times n}(\mathbb{R})$. In each situation, we also considered other bases besides the standard ones. After this, we proved the following **Theorem**: If $v_1, \ldots, v_n$ is a basis for V, then every vector $v \in \mathsf{V}$ can be **uniquely** expressed as $v = \lambda_1 v_1 + \cdots + \lambda_n v_n$ with each $\lambda_i \in \mathbb{F}$. This motivated the following observation: If we have a basis for V of length n, then to specify an arbitrary vector in V, we need only specify a list of n constants. After this, we proved the following **Theorem**: If $v_1, \ldots, v_n$ and $w_1, \ldots, w_m$ are two bases for V, then $m = n$. That is, all bases for V must have the same number of terms. This was a relatively easy application of the Replacement Lemma. This allowed us to make the following **Definition**: The *dimension* of V is the length of any basis for V. We ended by computing the dimension of the vector spaces that usually pop up in our examples.

Week 05 Tuesday 09/24 and Thursday 09/26

Read Axler 2A, start 2B

Practice Axler 2A: 3,5,6,8
FIS: Page 33, 3, 4, 5abcdef, Page 41 2cdef

Tuesday We recalled the concept of the span of vectors in a vector space. We discussed how determining whether a given vector lies in the span of others can be translated into a system of equations, and then solved using methods from MATH 290. After going over numerous examples of this, we defined a vector space V to be *finite-dimensional* if there exist vectors $v_1, \ldots, v_n$ with $\mathsf{V} = \text{span}(v_1, \ldots, v_n)$. In this case, we say that $v_1, \ldots, v_n$ *generate* V. We then covered many examples, producing multiple sets of generators for $\mathbb{R}^2$, and motivated by this, a canonical set of generators for $\mathbb{R}$. We also showed that P_n, the collection of all polynomials of degree at most n, is finite-dimensional by exhibiting a set of generators for this vector space. We also gave a proof, by contradiction (*reductio ad absurdum*), of the fact that P, the vector space of *all* polynomials (that is, without any explicit bound on the degree) is *not* finite-dimensional. We then introduced the concept of *linear independence*, and through examples, showed how to reduce the problem of determining whether vectors are linearly independent to a MATH 290 problem.

Thursday We started the lecture by going over a problem from the homework, that asked us to find a generating set for $\mathsf{W} = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$. After this, we recalled the concept of linear independence, and went over examples of how to determine whether given lists of vectors are independent or dependent. After this, we observed that a list consisting of a single vector is independent if and only if that vector is nonzero, that every vector in a list of independent vectors must be nonzero, and that any list obtained by omitting entries in an independent list of vectors is also an independent list of vectors. After that, we presented the following **Theorem**(Replacement Lemma) Suppose that V is finite-dimensional. If $x_1, \ldots, x_m$ are an arbitrary collection of independent vectors in V, and $y_1, \ldots, y_n$ is an arbitrary generating set for V, then $m \leq n$. Furthermore, we may omit a certain collection of m-many of the y's and replace them with $x_1, \ldots, x_m$, and in doing so, obtain a new generating set consisting of the remaining $n - m$ y's and all of the x's. *We did not prove this result yet.* Rather, due to time constraints, we presented some examples of how to use it.

Week 04 Tuesday 09/17 and Thursday 09/19

Read 2A

Practice 1C: 14, 15, 16, 20 2A: 1, 2, 3

Tuesday We spent the lecture considering sums of vector spaces. We started by extending this definition to allow for summing a finite number of subspaces, and proved that this is the smallest subspace containing each term in the sum. After going over more examples, we defined what it means to say that $\mathsf{W}_1 + \mathsf{W}_2 = \mathsf{W}_1 \oplus \mathsf{W}_2$, that is, that the sum of vector spaces is a *direct sum*, where $\mathsf{W}_1, \mathsf{W}_2$ are subspaces of V. We considered more examples of sums of vector spaces, both direct and not. We concluded by proving the following **Theorem**: If $\mathsf{W}_1, \mathsf{W}_2$ are subspaces of V, then TFAE.

(1) $\mathsf{W}_1 + \mathsf{W}_2 = \mathsf{W}_1 \oplus \mathsf{W}_2$.
(2) The only way to express $0 = x + y$ with $x \in \mathsf{W}_1, y \in \mathsf{W}_2$ is $x = y = 0$.
(3) $\mathsf{W}_1 \cap \mathsf{W}_2 = \{0\}$.

Thursday We started lecture by going over another example of how to compute the sum of two subspaces. After this, we had Quiz 02. After that, we began Chapter 2 by introducing the concept of a linear combination of vectors, and then used this to define the *span* of vectors. After going over examples, we proved the following **Theorem**: If $v_1, \ldots, v_n \in \mathsf{V}$, then $\operatorname{span}(v_1, \ldots, v_n)$ is the smallest subspace of V containing each of the vectors $v_1, \ldots, v_n$.

Week 03 Tuesday 09/10 and Thursday 09/12

Read Finish 1C, start 2A

Practice 1C: 1, 5, 6, 8, 10, 11, 14, 15, 16

Tuesday We recalled the conditions needed to check whether a subset W of a vector space V is a subspace. After this, we spent a lot of time going over examples of how to explicitly use these conditions to check if certain subsets of $\mathbb{R}^3$ and $\mathbb{R}^4$ were subspaces. After this, we proved that the intersection of two subspaces is always a subspace, and also produced an explicit example of a union of two subspaces that was not a subspace. Students will be asked to reconsider this in the assigned practice problems. In that problem, try to construct a different counterexample to illustrate that the union of subspaces need not be a subspace.

Thursday We recalled the basics of subspaces. Motivated by the fact that the union of two subspaces need not be a subspace, we introduced the notion of the sum of two subspaces of a fixed vector space. Recall the **Definition**: If Z, W are subspaces of a vector space V, then $\mathsf{Z} + \mathsf{W} = \{v + w : v \in \mathsf{V}, w \in \mathsf{W}\}$. This is a subspace of V, and in fact, the smallest subspace of V that contains W and W. We went over a bunch of examples of this operation on subspaces.

Week 02 Tuesday 09/03 and Thursday 09/05

Read 1B, 1C, start 2A

Practice 1A: 10, 14. 1B: 1, 2 1C: 1

Tuesday We started by proving the following theorem, which we regard as an improvement of the field axioms. **Theorem**: If $\mathbb{F}$ is a field and $\alpha \in \mathbb{F}$, then the multiplicative inverse of α is unique. Similarly, if $\beta \in \mathbb{F}$ is arbitrary, then the additive inverse of β is unique. After this, we recalled the definition of $\mathbb{F}^n$, and inspired by the specific cases of $\mathbb{R}^2, \mathbb{R}^3$ that we are familiar with from Calculus, we introduced the so-called *field axioms*. We then defined a vector space over a field $\mathbb{F}$ to be any set that satisfies these axions. We spent the rest of the class going over examples of vector spaces.

Thursday We started off by proving the following **Theorem**: Let V be a vector space over a field $\mathbb{F}$.

- The additive identity 0_{V} of V is unique.
- The additive inverse of a given element of V is unique.
- $0_{\mathbb{F}} \cdot x = 0_{\mathsf{V}}$ for every $x \in \mathsf{V}$.
- $\lambda \cdot 0_{\mathsf{V}} = 0_{\mathsf{V}}$ for every $\lambda \in \mathbf{F}$.
- $(-1) \cdot x = -x$, the additive inverse of x, for every $x \in \mathsf{V}$.

We next introduced the notion of a subspace of a given vector space, and presented the following **Theorem**: If W is a subset of V, then W is a subspace of V if and only if the following conditions are satisfied.

- W contains 0_{V}.
- If $x, y \in \mathsf{W}$, then $x + y \in \mathsf{W}$, i.e., W is closed under addition.
- If $x \in \mathsf{W}$ and $\lambda \in \mathbb{F}$, then $\lambda x \in \mathsf{W}$, i.e., W is closed under scalar multiplication.

We spent the rest of the class going over examples, e.g., we say that every line through the origin is a subspace of $\mathbb{R}^2$, and that every plane through the origin is a subspace of $\mathbb{R}^3$. We also considered some nice shapes that are not subspaces of $\mathbb{R}^3$, and discussed exactly what goes into establishing that a certain subset is *not* a subspace, using the above three conditions.

Week 01 Tuesday 08/27 and Thursday 08/29

Read 1A, 1B

Practice None this week

Tuesday Welcome to Math 590! We spent the first part of the lecture on the syllabus and introductions. We started by introducing some mathematical notation, including the symbols $\forall$ (for all), $\exists$ (there exists), $\in$ (is an element of, is contained in), basic set notation, $\mathbb{Z}$ (the integers, whole numbers), $\mathbb{Q}$ (the rational numbers), $\mathbb{R}$ (the real numbers), and $\mathbb{C}$ (the complex numbers). We observed that there are two basic types of algebraic objects in MATH 290/291, namely, scalars and vectors. We spent the rest of the class discussing scalars, gathered the nice properties of $\mathbb{R}$ in a list, and pondered to what extent these other sets satisfy analogous properties.

Special assignment: Read our syllabus, and introduce yourself to me via email. Do this ASAP, and by midnight on Friday at the latest. Consider including the following information. Of course, you can say less, or more.

- Your major/minor, reasons for registering, and goals for this course.
- Your math background.
- Any future aspirations involving math. For instance, do you plan to attend graduate school, or pursue a career, in a math-adjacent area?
- Any personal facts you would like to share. For example, I discussed my family, hometown, educational background, hobbies, and pets.
- Any personal circumstances that might impact your performance.
- If you are OK doing so, a recent photo of yourself.
  Pet photos are very welcome!
- Title your message `[math-590] Introduction`. Make certain that your message conforms to the email policies described in our syllabus.

Thursday Today's lecture was primarily concerned with the notion of fields. To start, we recalled a list of conditions that we later called the field axioms. The conditions in this list were inspired by the familiar rules that govern addition and multiplication of real numbers $\mathbb{R}$. We saw that that $\mathbb{Q}, \mathbb{R}, \mathbb{C}$ are each a field, and that $\mathbb{Z}$ is not a field (the problem being that the multiplicative inverse of an integer is usually not an integer). We also spent some time discussing *finite fields*. More precisely, we saw that the binary number system is a field, and constructed fields with 3 and 5 elements as well using modular arithmetic. After this, we introduced the set $\mathbb{F}^n$, which is the set consisting of all lists of length/size n with entries in $\mathbb{F}$. Of course, this is a simple generalization of the sets $\mathbb{R}^2, \mathbb{R}^3$ considered in Calc III.