Midterm 01 Conceptual Review

Exam details

Length	5-6 problems, many of which may have multiple parts.
Material	All topics covered so far this semester.
Aids	Both sides of one standard-sized (3×5 inch) notecard is allowed, but no calculators are allowed.

Note: Sample problems from Axler appear in **teal**, sample problems from FIS appear in **blue** and quizzes appear in **red**.

Vector spaces and subspaces

(1) In class, we've presented many different examples of vector spaces, e.g., $\mathbb{R}^n, \mathsf{P}_n, \mathsf{M}_{m\times n}$. Familiarize yourself with the definitions and notations for each of these.
(2) Can you prove some basic facts about an arbitrary vector space using only the axioms? **1B: 2** **1.2: Problem 9**
(3) What is the definition of a subspace?
(4) Exactly what conditions must be checked to guarantee that a subset W of a vector space is a subspace? Can you use these conditions, in practice, to verify or disprove that a specific set W is a subspace? **1C: 1** **1:3, 8, 10, 11** **Quiz 01**
(5) What is the sum of two vector spaces? Can you compute examples? **1C: 14, 15** **Quiz 02**

Linear combinations and independence

(1) What is the definition of a linear combination of vectors?
(2) If $x_1, \ldots, x_n \in \mathsf{V}$, what is the definition of $\operatorname{span}(x_1, \ldots, x_n)$? **Quiz 03**
(3) In what sense is $\operatorname{span}(x_1, \ldots, x_n)$ minimal?
(4) What does it mean to say that a list of vectors generates V?
(5) In a concrete setting, if you are given vectors $x_1, \cdots, x_n$ of a vector space, can you check if some given element $\mathbf{v}$ is in $\operatorname{span}(x_1, \cdots, x_n)$? For example, what linear system would you have to solve to do this? Can you actually solve such a system, or show that no solution exists? **2A: 2, 3** **1.4: 3, 4, 5**
(6) What does it mean to say that a collection of vectors is linearly independent?
1.5: 9
(7) Can you verify if a collection of vectors are linearly independent? **2A: 5, 6, 8** **1.5: 2, 3**

Bases, dimension, and subspaces

The last items from this section will be covered in the last lecture before Midterm 01. At that points, any topics not covered in lecture will be removed from this list.

(1) What does it mean to say that a list of vectors is a basis for V?
(2) What is the dimension of a vector space? What does it mean to be finite-dimensional?
(3) In class, we've gone over many examples of finite-dimensional vector spaces. In each such example, can you list a basis?
(4) In a concrete setting, can you verify whether a given set of vectors is a basis for a vector space? **1.6: 2, 3, 9**
(5) Under certain conditions, you can "skip" some steps when verifying that a certain candidate set is a basis. When can you do this?
(6) Do you understand how a basis for an abstract vector space can be used to build another basis? **1.6: 11, 12**
(7) Given an explicitly described subspace W of V, can you compute a basis, and its dimension? **1.6: 13, 14, 15, 16, 18, 26**