Midterm 02 Conceptual Review

Exam details

Length 5-6 problems, many of which may have multiple parts.
Material All topics covered so far this semester.
Aids Both sides of one standard-sized (3×5 inch) notecard is allowed, but no calculators are allowed.

Note: Problems from FIS appear in **blue** and quizzes appear in **red**.

Basics regarding linear transformations

(1) What is the definition of a linear transformation $\mathsf{T} : \mathsf{V} \to \mathsf{W}$? Can you verify whether a particular function is linear? **2.1: 2, 3, 4, 5, 8, 9 Quiz 4**
(2) What is the definition of the kernel and range of a linear transformation?
(3) How is $\ker(\mathsf{T})$ related to the property of T being 1-1?
(4) How is $\mathsf{R}(\mathsf{T})$ related to the property of T being onto?
(5) How are kernels and ranges related to subspaces?
(6) Given a linear transformation T, how does one compute its kernel? **2.1: 2, 3, 4, 5**
(7) Similarly, given a linear transformation T, how does one compute its range? Even further, how does one compute a basis for its range? **2.1: 2, 3, 4, 5**
(8) What is the extrapolation principle? Can you use it to determine the values of linear transformations at particular inputs? Can you use it to determine if a particular linear transformation is 1-1 or onto? **2.1: 10, 11, 12 Quiz 6**

Rank-Nullity Theorem, and applications

(1) What is the precise statement of the Rank-Nullity Theorem?
(2) Can you apply this theorem to determine if a given linear transformation is 1-1? **Quiz 5 2.1: 2, 3, 4, 5, 10**
(3) Can you apply this theorem to determine if a given linear transformation is onto? **Quiz 5 2.1: 2, 3, 4, 5**

Linear transformations and matrices

(1) Given a linear transformation $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ and ordered bases β for V and γ for W, what is the definition of the matrix $[\mathsf{T}]_\beta^\gamma$? **2.2: 2, 3, 4, 5 Quiz 6**

(2) What does composition of linear transformations correspond to for matrices? How does this simplify describing the composition of linear transformations? **2.3: 3a**
(3) Can you produce an explicit formula for T from the matrix $[\mathsf{T}]_\beta^\gamma$? To be emphasized during the last lecture before Midterm 02
(4) If T is invertible, then what is the inverse of the matrix $[\mathsf{T}]_\beta^\gamma$? If $\mathsf{T} : \mathsf{V} \to \mathsf{W}$ is an isomorphism and $\dim(\mathsf{V}) = \dim(\mathsf{W}) = 2$, then can you use the inverse of the matrix $[T]_\beta^\gamma$ to give an explicit formula for T^{-1}? **2.4: 1a** To be emphasized during the last lecture before Midterm 02